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Adaptive reference governor for constrained linear systems

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Abstract

This paper presents a novel adaptive reference governor for robust tracking control in constrained linear systems with bounded disturbances. The proposed reference governor achieves a performance improvement over the existing reference governors by virtue of its added feature: adaptability. The design of such an adaptive reference governor involves a nonlinear non-deterministic polynomial time (NP)-hard optimization problem because the solution of the optimization problem must be searched for the infinite number of sequences of disturbance. The SDP relaxation method turns out to allow the nonlinear NP-hard problem to be recast into an SDP, which may be readily solved in polynomial time.

Keywords: Adaptability; Reference governor; Constrained linear system; NP-hard problem; SDP relaxation

1. Introduction

A constrained control problem considers dynamical systems with pointwise-in-time constraints on control input and/or state variables, which is often encountered in practice [1]. The constraints may drive a closed-loop system unstable, which would be stable otherwise. One of the most effective ways to control such constrained systems is to adopt a so-called reference governor. Fig. 1 shows a typical block diagram of the reference governors. The role of the reference governor is to tailor the reference command y_c so that the control input and/or state variables always stay within the given constraints for all reference inputs and disturbances without sacrificing the performance of the overall system [2-4, 7, 9-11].

As in the model-based predictive control systems [12, 13, 15, 16, 21], the reference governor generates the future states based on the given model, using the current state, reference input, and disturbance along with its bound. The bounded time-varying disturbance renders each of the predicted future states into a bounded set as shown in Fig. 2. Equipped with the future states, the reference governor generates a sequence of governor outputs that guarantees the control input and/or state variables to stay within the given constraints at all of the future time steps for any possible disturbances. Hereinafter, this type of a reference governor will be called as a *robust* reference governor in the sense that it seeks for a solution that holds for all uncertainties. It should be noted here that such robustness could be achieved at the expense of performance.

This paper presents a novel *adaptive* reference governor for the robust tracking control of constrained linear time-invariant systems with bounded time-varying disturbances. The goal of the proposed *adaptive* reference governor is to achieve robustness against the disturbances without sacrificing the performance. The proposed reference governor is adaptive in the sense that a governor output for a certain future time step reflects on the past trace of the disturbances at the current time step. For example, the

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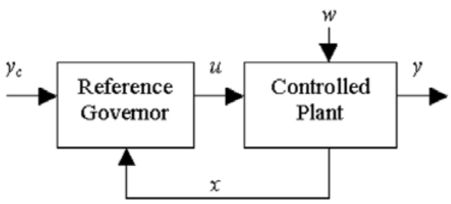

Fig. 1. Typical reference governor.

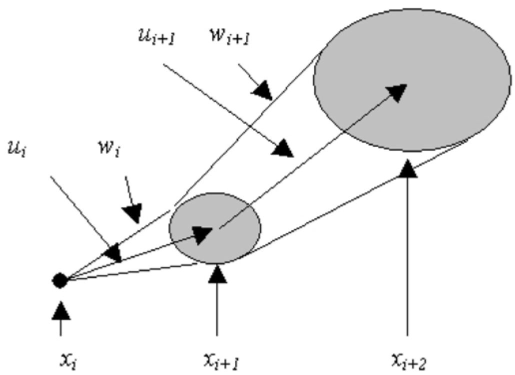

Fig. 2. Uncertainty evolution.

governor output for the next time step is given as a function of the disturbance between the current and the next time steps, which is not known at the current time step but its bound would be known by the next time step. Hence, at the next time step, the reference governor may adapt its output based on the now known bound of the disturbance.

The difficulty in realizing the *adaptability* lies in the non-deterministic polynomial time (NP)-hard property of the given problem, which is shown in the following section. To circumvent such a difficulty, this paper recasts the given problem into an adaptive semi-definite program (ASDP) [19, 20] and, in turn, relaxes the ASDP into an SDP [5, 22] via the *S*-procedure [5, 8, 23]. SDPs are convex optimization problems that are proven to be solvable in polynomial time [17, 22].

2. Problem formulation

The controlled process is assumed to be asymptotically stable, represented by

$$\boldsymbol{x}_{i+1} = \boldsymbol{A}\boldsymbol{x}_i + \boldsymbol{b}_u + \boldsymbol{b}_w w_i \tag{1}$$

$$y_i = \boldsymbol{c}^T \boldsymbol{x}_i + du_i + d_w w_i, \tag{2}$$

where $\boldsymbol{x}_i \in R^{n_x}$ is the state vector, $u_i \in R$ is the attenuated reference command, $y_i \in R$ is the system output, and $w_i \in R$ is the disturbance with the bound

$$|w_i| \le w_i^*, \quad i = 0, 1, 2, \cdots. \tag{3}$$

Here, w_i^* is known *a priori*. It is further assumed that at the i^{th} time step, w_i is known up to the time step $i-1$.

The linear constraints on control input and/or states may be generally expressed as

$$\left|\boldsymbol{e}_j^T \boldsymbol{x}_i + f_j u_i + g_j w_i - \overline{p}_{ji}\right| \le p_{ji}^*, \quad \forall j = 1, \cdots, n_c, \ \forall i = 0, 1, \cdots. \tag{4}$$

where n_c is the total number of constraints, and $\boldsymbol{e}_j$, f_j, g_j, $\overline{p}_{ji}$ and p_{ji}^* are vectors of appropriate dimensions. Note that the above inequalities may represent a wide class of constraints on state variables, input, input rate, output overshoot and undershoot, etc.

For simplicity, starting from the 0^{th} time step, consider how to calculate the 1^{st} plant input. That is, given the initial state $\boldsymbol{x}_0$ and plant input u_0 (typically, $u_0 = 0$) that satisfy Eq. (4) for $i = 0$, an optimal plant input u_1 is sought after in the sense that

Eq. (4) is satisfied from the time step 1 to n_u (the horizon size). The n_u-step control sequence

$$\boldsymbol{u} = \begin{bmatrix} u_1 & \cdots & u_{n_u} \end{bmatrix}^T, \tag{5}$$

and any allowable n_u-step disturbance sequence (allowed by Eq. (3))

$$\boldsymbol{w} = \begin{bmatrix} w_0 & \cdots & w_{n_u} \end{bmatrix}^T. \tag{6}$$

The same control sequence $\boldsymbol{u}$ minimizes the cost function

$$J = \sum_{i=1}^{n_u} |y_c - y_i|^2, \tag{7}$$

where y_c is the set-point reference command.

Minimizing Eq. (7) leads to fast convergence to the set-point reference command, which is one of the most common control goals. Note that the problem can be generalized to a tracking problem with y_{c_i} instead of y_c for $i = 1, \cdots, n_u$.

Before proceeding, define a set of allowable disturbance sequences, $\boldsymbol{W}_0$, as

$$\boldsymbol{W}_0 \triangleq \left\{(z_0,\cdots z_{n_u})^T \mid |z_i| \le w_i^*, i = 0,\cdots,n_u\right\}. \tag{8}$$

Then, the problem discussed thus far may be succinctly formulated as

P_0 : *For given* $\boldsymbol{x}_0$ and u_0, *find* $\boldsymbol{u}$ *minimizing the cost* J *given by Eq. (7), while, for any* $\boldsymbol{w} \in \boldsymbol{W}_0$, *the constraint given by Eq. (4) is satisfied for* $i = 1,\cdots,n_u$.

When solving P_0, note that the i^{th} input u_i must be chosen so that Eq. (4) is satisfied for all past disturbance w_j, $j = 0,\cdots,i-1$ satisfying Eq. (3) as well as the i^{th} disturbance w_i, granted that w_j, $j = 0,\cdots,i-1$ would be known by the i^{th} time step. In this sense, P_0 is a robust predictive control problem. Ignoring the computational load, a larger n_u would result in better performance. However, it is easily observed from Fig. 2 that the feasibility of P_0 is greatly influenced by n_u. That is, as n_u becomes larger, it is less likely that P_0 is feasible.

Although performance degradation may be encountered from time to time, the problem with the feasibility of P_0 may be attacked by introducing *adaptability*, *i.e.,* by stipulating that u_i be chosen to be a function of w_j, $j = 0,\cdots,i-1$. First, the solution of P_0 is parameterized as a linear combination of disturbance independent ($\boldsymbol{u}_a$) and dependent terms ($\boldsymbol{U}_b\boldsymbol{w}$):

$$\boldsymbol{u} = \boldsymbol{u}_a + \boldsymbol{U}_b\boldsymbol{w}, \tag{9}$$

Where

$$\boldsymbol{u}_a = \begin{bmatrix} u_{a1} & \cdots & u_{an_u} \end{bmatrix}^T \tag{10}$$

$$\boldsymbol{U}_b = \begin{bmatrix} u_{b11} & 0 & \cdots & 0 & 0 \\ u_{b21} & u_{b22} & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ u_{bn_u1} & u_{bn_u2} & \cdots & u_{bn_un_u} & 0 \end{bmatrix} \tag{11}$$

It is easily seen in Eq. (9) that the control command at each time step becomes the output of a linear causal filter with the disturbance w_i as an input:

$$u_i = u_{ai} + \sum_{k=1}^{i} u_{bik} w_{k-1}, \quad i = 1,\cdots,n_u. \tag{12}$$

Although the parameterization in Eq. (9) may still cause a feasibility problem, the branch and bound algorithm can help to tackle it. With Eq. (9), P_0 may be recast into

P_1 : *For given* $\boldsymbol{x}_0$ and u_0, *find* $\boldsymbol{u}_a$ and $\boldsymbol{U}_b$ *minimizing the cost* J *given by Eq. (7), while, for any* $\boldsymbol{w} \in \boldsymbol{W}_0$, *the constraint given by Eq. (4) is satisfied for* $i = 1,\cdots,n_u$.

P_1 may be thought of as an *adaptive* predictive problem in that u_i may be found for each sequence { w_j, $j = 0,\cdots,i-1$ } among an infinite number of sequences satisfying Eq. (3) for $j = 0,\cdots,i-1$. However, note that P_1 is now an NP-hard problem because of the infinite number of sequences { w_j, $j = 0,\cdots,i-1$ }, $i = 1,\cdots,n_u$, which implies that adaptability cannot be realized in practice. In the following section, the SDP relaxation method helps to recast P_1 into a convex problem. The SDP relaxation method has recently expanded its territory by providing satisfactory solutions in many applications, where the solution to the relaxed problem turns out to e very close to the optimal one [6, 14, 18].

3. Numerical solution via SDP relaxation

In this section, the SDP relaxation method helps to recast P_1 into a convex optimization problem. More specifically, the SDP relaxation method achieves its goal by replacing w_i with w_i^*, for $i = 0,\cdots,n_u$.

First, the cost function is relaxed. The cost function in Eq. (7) is converted into the following constraint:

$$\gamma \ge \sum_{i=1}^{n_u} |y_c - y_i|^2, \tag{13}$$

where γ is a new control variable. It is obvious that minimizing γ while satisfying Eq. (13) is equivalent to minimizing J. Algebraic manipulations with Eq. (1) result in

$$\boldsymbol{x}_{i+1} = \boldsymbol{h}_i + \boldsymbol{F}_i\boldsymbol{u}_a + (\boldsymbol{F}_i\boldsymbol{U}_b + \boldsymbol{G}_i)\boldsymbol{w}, \ \forall i = 1,2,\cdots, \tag{14}$$

where

$$\boldsymbol{h}_i = \boldsymbol{A}^{i-1}(\boldsymbol{A}x_0 + \boldsymbol{b}u_0) \tag{15}$$

$$\boldsymbol{F}_i = \begin{cases} \boldsymbol{O}^{n_x \times n_u}, & \text{If } i = 1 \\ \left[\boldsymbol{A}^{i\text{-}2}\boldsymbol{b} \cdots \boldsymbol{A}\boldsymbol{b}\ \boldsymbol{b}\ \boldsymbol{O}^{n_x \times (n_u - i + 1)}\right], & \text{otherwise} \end{cases} \tag{16}$$

$$\boldsymbol{G}_i = \left[\boldsymbol{A}^{i\text{-}1}\boldsymbol{b}_w \cdots \boldsymbol{A}\boldsymbol{b}_w\ \boldsymbol{b}_w\ \boldsymbol{O}^{n_x \times (n_u - i + 1)}\right]. \tag{17}$$

Here, $\boldsymbol{O}^{n \times m}$ is an $n \times m$ matrix whose elements are all zeros. $\boldsymbol{J}_i$, $i=1,\cdots,n_u$ is an $n_u \times 1$ vector whose elements are all zeros except $\boldsymbol{J}_i(i)=1$ while $\boldsymbol{K}_i$, $i=0,\cdots,n_u$ is an $(n_u+1)\times 1$ vector whose elements are all zeros except $\boldsymbol{K}_i(i+1)=1$. Then, by Eq. (1) and Eq. (14), the constraint in Eq. (13) may be rephrased as

$$\gamma \geq r_0 + \boldsymbol{u}_a^T \boldsymbol{P}_{r0} \boldsymbol{u}_a + 2\boldsymbol{q}_{r0}^T \boldsymbol{u}_a + 2(\boldsymbol{q}_{w0}^T + \boldsymbol{q}_{r0}^T \boldsymbol{U}_b)\boldsymbol{w} + 2\boldsymbol{w}^T(\boldsymbol{U}_b \boldsymbol{P}_{r0} + \boldsymbol{Q}_0)\boldsymbol{u}_a + \boldsymbol{w}^T(\boldsymbol{P}_{w0} + \boldsymbol{U}_b^T \boldsymbol{P}_{r0} \boldsymbol{U}_b + \boldsymbol{Q}_0 \boldsymbol{U}_b + \boldsymbol{U}_b^T \boldsymbol{Q}_0^T)\boldsymbol{w} \tag{18}$$

where

$$r_0 = \sum_{i=1}^{n_u} \left| y_c - \boldsymbol{c}^T \boldsymbol{h}_i \right|^2 \tag{19}$$

$$\boldsymbol{P}_{r0} = \sum_{i=1}^{n_u} (\boldsymbol{c}^T \boldsymbol{F}_i + d\boldsymbol{J}_i^T)^T (\boldsymbol{c}^T \boldsymbol{F}_i + d\boldsymbol{J}_i^T) \tag{20}$$

$$\boldsymbol{P}_{w0} = \sum_{i=1}^{n_u} (\boldsymbol{c}^T \boldsymbol{G}_i + d_w \boldsymbol{K}_i^T)^T (\boldsymbol{c}^T \boldsymbol{G}_i + d_w \boldsymbol{K}_i^T) \tag{21}$$

$$\boldsymbol{q}_{r0} = -\sum_{i=1}^{n_u} (y_c - \boldsymbol{c}^T \boldsymbol{h}_i)(\boldsymbol{c}^T \boldsymbol{F}_i + d\boldsymbol{J}_i^T)^T \tag{22}$$

$$\boldsymbol{q}_{w0} = -\sum_{i=1}^{n_u} (y_c - \boldsymbol{c}^T \boldsymbol{h}_i)(\boldsymbol{c}^T \boldsymbol{G}_i + d_w \boldsymbol{K}_i^T)^T \tag{23}$$

$$\boldsymbol{Q}_0 = \sum_{i=1}^{n_u} (\boldsymbol{c}^T \boldsymbol{G}_i + d_w \boldsymbol{K}_i^T)^T (\boldsymbol{c}^T \boldsymbol{F}_i + d\boldsymbol{J}_i^T) \tag{24}$$

The constraint in Eq. (18), along with the Schur complement inequalities [5] becomes

$$\begin{bmatrix} \phi_0 \\ \psi \end{bmatrix}^T \begin{bmatrix} \xi_1 & \xi_2^T \\ \xi_2 & \boldsymbol{I}_{n_u} \end{bmatrix} \begin{bmatrix} \phi_0 \\ \psi \end{bmatrix} \geq 0, \ \forall \begin{bmatrix} \phi_0 & \psi^T \end{bmatrix}^T \in R^{(n_u+1)\times 1} \tag{25}$$

where ξ_1 and ξ_2 are temporary variables with an appropriate dimension, defined as

$$\xi_1 = \gamma - r_0 - 2\boldsymbol{q}_{r0}^T \boldsymbol{u}_a - 2(\boldsymbol{q}_{w0}^T + \boldsymbol{q}_{r0}^T \boldsymbol{U}_b + \boldsymbol{u}_a^T \boldsymbol{Q}_0^T)\boldsymbol{w} - \boldsymbol{w}^T(\boldsymbol{P}_{w0} + \boldsymbol{Q}_0 \boldsymbol{U}_b + \boldsymbol{U}_b^T \boldsymbol{Q}_0^T)\boldsymbol{w} \tag{26}$$

$$\xi_2 = \sqrt{\boldsymbol{P}_{r0}}\boldsymbol{u}_a + \sqrt{\boldsymbol{P}_{r0}}\boldsymbol{U}_b \boldsymbol{w} \tag{27}$$

Note that ξ_k denotes a temporary variable throughout this paper.

Let $\phi_i = w_{i-1}\phi_0$ and $\Phi = \begin{bmatrix} \phi_1 \cdots \phi_{n_u+1} \end{bmatrix}^T$. Then, Eq. (25) can be shown to be equivalent to the following inequality,

$$\phi_0^2(\gamma - r_0 - 2\boldsymbol{q}_{r0}^T \boldsymbol{u}_a) - 2\phi_0(\boldsymbol{q}_{w0}^T + \boldsymbol{q}_{r0}^T \boldsymbol{U}_b + \boldsymbol{u}_a^T \boldsymbol{Q}_0^T)\Phi - \Phi^T(\boldsymbol{P}_{w0} + \boldsymbol{Q}_0 \boldsymbol{U}_b + \boldsymbol{U}_b^T \boldsymbol{Q}_0^T)\Phi + 2\phi_0 \boldsymbol{u}_a^T \sqrt{\boldsymbol{P}_{r0}}\Psi + 2\Phi^T \boldsymbol{U}_b^T \sqrt{\boldsymbol{P}_{r0}}\Psi + \Psi^T \boldsymbol{I}_{n_u}\Psi \geq 0 \tag{28}$$

for all $\begin{bmatrix} \phi_0 & \Phi^T & \psi^T \end{bmatrix}^T \in R^{(2n_u+2)\times 1}$ such that

$$\phi_i^2 \leq {w_{i-1}^*}^2 \phi_0^2 \tag{29}$$

Now, the S-procedure along with slack variables $s_{0i} \geq 0$, $i=1,\cdots,(n_u+1)$ leads to the following sufficient condition for Eq. (28)

$$\phi_0^2(\gamma - r_0 - 2\boldsymbol{q}_{r0}^T \boldsymbol{u}_a) - 2\phi_0(\boldsymbol{q}_{w0}^T + \boldsymbol{q}_{r0}^T \boldsymbol{U}_b + \boldsymbol{u}_a^T \boldsymbol{Q}_0^T)\Phi - \Phi^T(\boldsymbol{P}_{w0} + \boldsymbol{Q}_0 \boldsymbol{U}_b + \boldsymbol{U}_b^T \boldsymbol{Q}_0^T)\Phi + 2\phi_0 \boldsymbol{u}_a^T \sqrt{\boldsymbol{P}_{r0}}\Psi + 2\Phi^T \boldsymbol{U}_b^T \sqrt{\boldsymbol{P}_{r0}}\Psi + \Psi^T \boldsymbol{I}_{n_u}\Psi - \sum_{i=1}^{n_u+1} s_{0i}({w_{i-1}^*}^2 \phi_0^2 - \phi_i^2) \geq 0 \tag{30}$$

for all $\begin{bmatrix} \phi_0 & \Phi^T & \psi^T \end{bmatrix}^T \in R^{(2n_u+2)\times 1}$.

Eq. (30) may be compactly represented as the following LMI:

$$\begin{bmatrix} \xi_3 & \xi_4 & \boldsymbol{u}_a^T \sqrt{\boldsymbol{P}_{r0}} \\ \xi_4^T & \xi_5 & \boldsymbol{U}_b^T \sqrt{\boldsymbol{P}_{r0}} \\ \sqrt{\boldsymbol{P}_{r0}}\boldsymbol{u}_a & \sqrt{\boldsymbol{P}_{r0}}\boldsymbol{U}_b & \boldsymbol{I}_{n_u} \end{bmatrix} \geq 0 \tag{31}$$

$$s_{0i} \geq 0, \ \forall i = 1,\cdots,(n_u+1) \tag{32}$$

where

$$\xi_3 = \gamma - r_0 - 2\boldsymbol{q}_{r0}^T \boldsymbol{u}_a - \sum_{i=1}^{n_u+1} {w_{i-1}^*}^2 s_{0i} \tag{33}$$

$$\xi_4 = -(\boldsymbol{q}_{w0}^T + \boldsymbol{q}_{r0}^T \boldsymbol{U}_b + \boldsymbol{u}_a^T \boldsymbol{Q}_0^T) \tag{34}$$

$$\xi_5 = \mathrm{diag}([s_{01} \cdots s_{0n_u+1}]) - (\boldsymbol{P}_{w0} + \boldsymbol{Q}_0 \boldsymbol{U}_b + \boldsymbol{U}_b^T \boldsymbol{Q}_0^T) \tag{35}$$

It must be noted that $\boldsymbol{w}$ in Eq. (18) is replaced with w_i^*, $i=0,\cdots,n_u$ in Eq. (31).

Now, consider the constraint Eq. (4) together with Eq. (9). By using Eqs. (15)- (17) and the Schur complement, Eq. (4) may be compactly described as

$$\begin{bmatrix} \phi_0 \\ \varphi \end{bmatrix}^T \begin{bmatrix} p_{ji}^{*2} & \xi_6 \\ \xi_6 & 1 \end{bmatrix} \begin{bmatrix} \phi_0 \\ \varphi \end{bmatrix} \geq 0, \ \forall [\phi_0 \quad \varphi]^T \in R^{2\times 1} \quad (36)$$

where

$$\xi_6 = \boldsymbol{e}_j^T \boldsymbol{h}_i + (\boldsymbol{e}_j^T \boldsymbol{F}_i + f_j \boldsymbol{J}_i^T)\boldsymbol{u} + (\boldsymbol{e}_j^T \boldsymbol{G}_i + g_j \boldsymbol{K}_i^T)\boldsymbol{w} - \bar{p}_{ji} \quad (37)$$

In turn, Eq. (36) may be recast as

$$\phi_0^2 p_{ji}^{*2} + \varphi^2 + 2[\boldsymbol{e}_j^T \boldsymbol{h}_i + (\boldsymbol{e}_j^T \boldsymbol{F}_i + f_j \boldsymbol{J}_i^T)\boldsymbol{u}_a - \bar{p}_{ji}]\phi_0\varphi + 2[(\boldsymbol{e}_j^T \boldsymbol{F}_i + f_j \boldsymbol{J}_i^T)\boldsymbol{U}_b + (\boldsymbol{e}_j^T \boldsymbol{G}_i + g_j \boldsymbol{K}_i^T)]\Phi\varphi \geq 0 \quad (38)$$

for all $\left[\phi_0 \ \Phi^T \ \varphi^T\right]^T \in R^{(n_u+3)\times 1}$ such that

$$\phi_i^2 \leq w_{i-1}^{*\,2} \phi_0^2 \quad (39)$$

Then, the S-procedure along with slack variables $s_{ji} \geq 0$, $i = 1, \cdots, (n_u + 1)$ leads to a sufficient condition for Eq. (38) to hold:

$$\begin{aligned} &\phi_0^2 p_{ji}^{*2} + \varphi^2 + 2[\boldsymbol{e}_j^T \boldsymbol{h}_i + (\boldsymbol{e}_j^T \boldsymbol{F}_i + f_j \boldsymbol{J}_i^T)\boldsymbol{u}_a - \bar{p}_{ji}]\phi_0\varphi \\ &+2[(\boldsymbol{e}_j^T \boldsymbol{F}_i + f_j \boldsymbol{J}_i^T)\boldsymbol{U}_b + (\boldsymbol{e}_j^T \boldsymbol{G}_i + g_j \boldsymbol{K}_i^T)]\Phi\varphi \\ &-\sum_{i=1}^{n_u+1} s_{ji}(w_{i-1}^{*\,2}\phi_0^2 - \phi_i^2) \geq 0, \\ &\forall \left[\phi_0 \ \Phi^T \ \varphi^T\right]^T \in R^{(n_u+3)\times 1} \end{aligned} \quad (40)$$

Now, Eq. (40) may be turned into the following LMI:

$$\begin{bmatrix} \xi_7 & \boldsymbol{O}^{1\times(n_u+1)} & \xi_8 \\ \boldsymbol{O}^{(n_u+1)\times 1} & \boldsymbol{S}_j & \xi_9^T \\ \xi_8 & \xi_9 & 1 \end{bmatrix} \geq 0 \quad (41)$$

where

$$\boldsymbol{S}_j = \mathrm{diag}([s_{j1} \cdots s_{jn_u+1}]) \quad (42)$$

$$\xi_7 = p_{ji}^{*2} - \sum_{i=1}^{n_u+1} w_{i-1}^{*\,2} s_{ji} \quad (43)$$

$$\xi_8 = \boldsymbol{e}_j^T \boldsymbol{h}_i + (\boldsymbol{e}_j^T \boldsymbol{F}_i + f_j \boldsymbol{J}_i^T)\boldsymbol{u}_a - \bar{p}_{ji} \quad (44)$$

$$\xi_9 = (\boldsymbol{e}_j^T \boldsymbol{F}_i + f_j \boldsymbol{J}_i^T)\boldsymbol{U}_b + (\boldsymbol{e}_j^T \boldsymbol{G}_i + g_j \boldsymbol{K}_i^T) \quad (45)$$

Note that Eq. (41) does not contain $\boldsymbol{w}$.

Now, it is possible to recast the original problem P_1 into an SDP. The original problem P_1 is feasible if the following SDP P_2 is feasible:

P_2 : *For given* $\boldsymbol{x}_0$ and u_0, *find* $\boldsymbol{u}_a$ *and* $\boldsymbol{U}_b$, $s_{0i} \geq 0$, $i = 1, \cdots, (n_u + 1)$, *and* $s_{ji} \geq 0$, $j = 1, \cdots, n_c$, $i = 1, \cdots, (n_u + 1)$, *such that* γ *is minimized and Eqs. (31), (32), and (41) hold for* $i = 1, \cdots, (n_u + 1)$.

Note that SDPs are convex optimization problems and are proven to be solvable in polynomial time [6, 14, 18]. Although it may be argued that the computational burden on solving P_2 limits the applicability of the proposed approach, the advance in the computer technology makes it possible to solve P_2 in real time in many practical problems.

It is worth noting that P_2 is more likely to be feasible as the bound of disturbance becomes smaller. Thus, even though P_2 is not feasible with respect to the current set of disturbance bounds, it is possible to find a finite covering of disturbances such that P_2 is feasible with respect to every element of the covering. One of the best ways to find such a covering may be to apply the so-called branch-and-bound-like algorithm explained as in [20].

4. Numerical example

Consider a simplified inverted pendulum problem introduced in [9]. The equation of motion is represented by

$$\ddot{\theta} - \theta = v \quad (46)$$

where θ is the angle between vertical line and the pendulum while v is the motor current, which is bounded as

$$|v| \leq 0.1 \quad (47)$$

A controller has been designed so that the whole system may be asymptotically stable (without the input constraint):

$$v = -[6 \ 2]\boldsymbol{x} + 5u \quad (48)$$

Yet, due to a measurement disturbance, w_i, the control input, v, is given by

$$v = -[6 \ 2]\boldsymbol{x} + 5u - 6w \quad (49)$$

where

$$w(t) = w^{*}\text{sign}(\sin(2t)) \tag{50}$$

The whole system is digitized with a sampling period $T = 0.1$ sec to give

$$\boldsymbol{x}_{i+1} = \begin{bmatrix} 0.97668 & 0.08988 \\ -0.44941 & 0.79692 \end{bmatrix} \boldsymbol{x}_i + \begin{bmatrix} 0.02332 \\ 0.44941 \end{bmatrix} u_i - \begin{bmatrix} 0.02798 \\ 0.53929 \end{bmatrix} w_i \tag{51}$$

$$y_i = \begin{bmatrix} 1 & 0 \end{bmatrix} \boldsymbol{x}_i \tag{52}$$

where y_i is the angle at the i^{th} time step and $w_i = w^{*}\text{sign}(\sin(2\times i))$, $i = 0, 1, 2, \cdots$. The constraint on v is now represented as

$$\left|-[6\ 2]\boldsymbol{x}_i + 5u_i - 6w_i\right| \le 0.1,\ \forall i = 0, 1, 2, \cdots \tag{53}$$

An *adaptive* reference governor is designed by solving P_2 in the previous section, along with the above system equations, while a *robust* reference governor is designed for comparison. A *robust* reference governor may be readily designed by using P_2 with $\boldsymbol{U}_b$ =0. Computer simulations have been performed for the following two cases:

$$y_c = 0.04,\ n_u = 15,\ w^{*}=0.002 \tag{54}$$

$$y_c = 0.04,\ n_u = 15,\ w^{*}=0.004 \tag{55}$$

Since there is only one constraint on the control input v, $n_c = 1$. The first step to solve P_2 is to calculate the LMI components in Eqs. (31), (32) and (41) such as $\boldsymbol{h}_i$, $\boldsymbol{F}_i$, $\boldsymbol{G}_i$, $\boldsymbol{P}_{r0}$, $\boldsymbol{P}_{w0}$, $\boldsymbol{q}_{r0}$, $\boldsymbol{q}_{w0}$ and $\boldsymbol{Q}_0$, respectively. Recall that the dimensions of $\boldsymbol{u}_a$ and $\boldsymbol{U}_b$ as well as those components are determined by the horizon size n_u. The simulation is performed by using the software SDPpack [24].

The simulation results are depicted in Figs. 3-10. First, Figs. 6 and 10 show that the input, v, is saturated at most time steps till 30 sec without a governor. As a result, the plant outputs without a governor display large overshoots as shown in Figs. 3 and 7. Figs. 3 and 7 also show that the disturbance leads to huge overshoots and steady state errors without a governor, while both *adaptive* and *robust* reference governors provide better set-point tracking performance.

It is obvious from Figs. 3, 4, 7, and 8 that the *adaptive* reference governor outperforms the *robust* reference governor. Especially, the *adaptive* reference governor shows less overshoot, smaller steady state error, and faster convergence than the *robust* reference governor. In addition, Figs. 5 and 9 indicate that

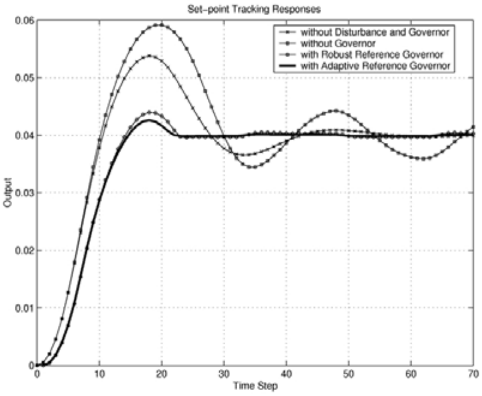

Fig. 3. Plant outputs (w^{*}=0.002).

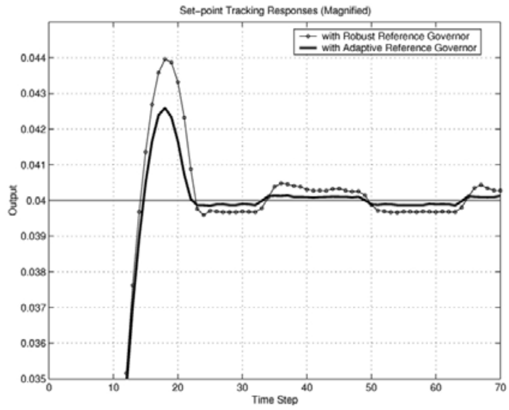

Fig. 4. Magnified plant outputs (w^{*}=0.002).

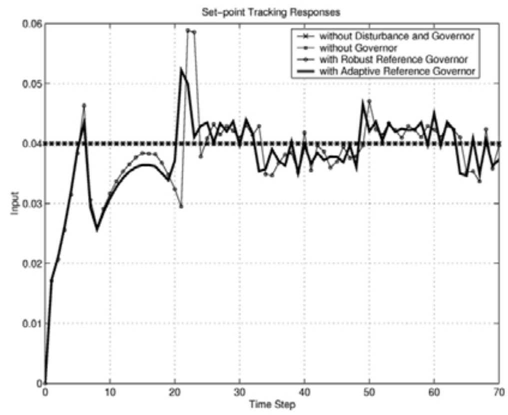

Fig. 5. Plant inputs (w^{*}=0.002).

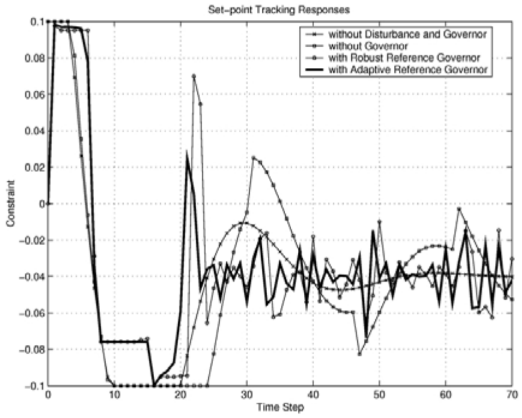

Fig. 6. Constraint (w^*=0.002).

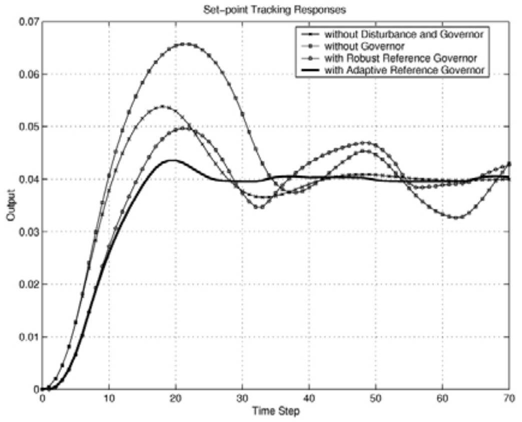

Fig. 7. Plant outputs (w^*=0.004).

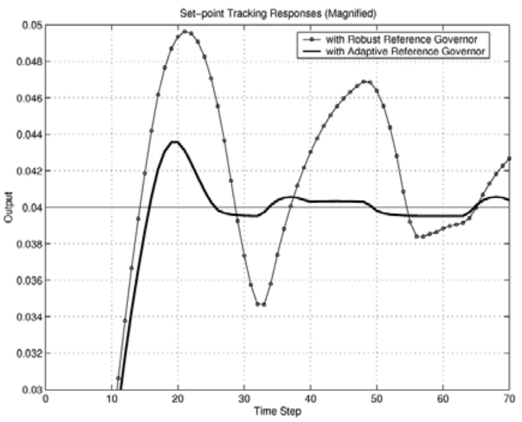

Fig. 8. Magnified plant outputs (w^*=0.004).

the plant input of the *adaptive* reference governor consumes less energy than that of the *robust* reference governor.

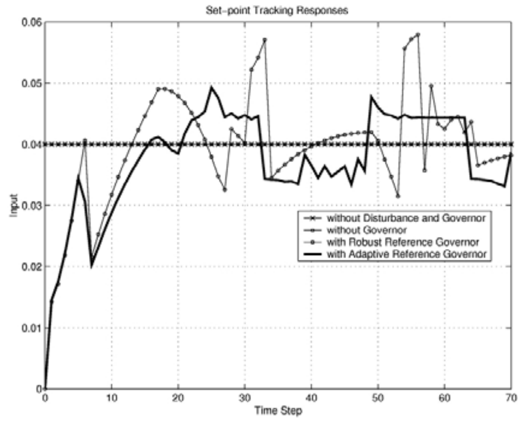

Fig. 9. Plant inputs (w^*=0.004).

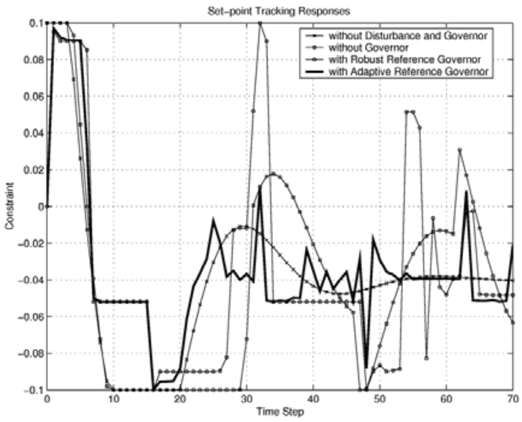

Fig. 10. Constraint (w^*=0.004).

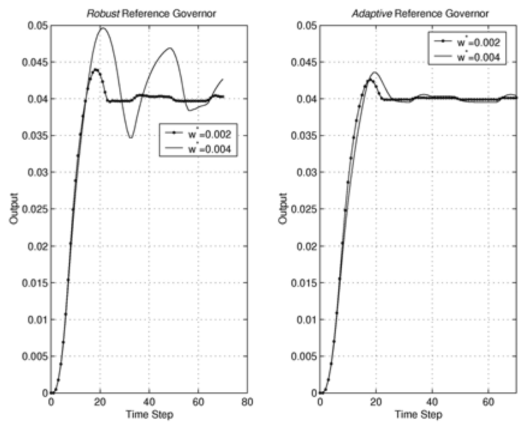

Fig. 11. Plant output variations for w^*=0.002 and w^*=0.004 .

Fig. 11 shows that the *adaptive* reference governor provides more robust performance against the disturbance than the *robust* reference governor does. That is, the output profiles of the *adaptive* reference governor for w^*=0.002 and w^*=0.004 remain almost unchanged, while those of the *robust* reference governor differ significantly from each other. Such a discrepancy is not unexpected since, in the case of the *robust* reference governor, a bigger disturbance bound leads to more conservative constraints and, in turn, performance degradation. On the other hand, the *adaptive* reference governor reduces the effect of such a variation on the disturbance bound via adaptation, and hence, the governor may provide much more improved performance against a disturbance bound variation.

5. Concluding remarks

The well-known SDP relaxation method is utilized to design an *adaptive* reference governor for the robust tracking control of a constrained linear system with bounded disturbance. The *adaptive* reference governors are shown to outperform the existing *robust* reference governors. Moreover, the optimization problem in the model-based predictive control scheme turns out more likely to be feasible by adopting *adaptability*. Although lacking in a thorough treatment on the feasibility and stability, the proposed approach has the potential to make an impact on many practical constrained control problems.

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